THE

A B C

OF

GEOMETRY.

SECTION A,

PART

OF

RATIONAL MATHEMATICS.

1895.

PROSPECTUS.

The New Geometry and Commensurational Arithmetic is now under way to be published.

The complete work is divided into three sections: *A*, *B*, *C*.

Section *A* is composed of three parts, one of which is now issued as the A B C of Geometry. The other two will be contained in one binding, and due notice will be given subscribers to the first part, when issued.

Section *B* is composed of four parts, all in one binding.

Section *C* is composed of five parts, all contained in one binding.

25 cents in stamps, by mail, will pay for a copy of the first part, sent to any address prepaid.

10 cents in stamps will pay for a complete and exhaustive exposition of the new, true "pi" value and its functions.

Address,

CHAS. DE MEDICI,
60 West 22d St., N. Y.

MEDICI'S

RATIONAL MATHEMATICS.

SECTION A

GEOMETRY

PART I.

FIRST PRINCIPLES AND PRIMARY ELEMENTS

TAUGHT BY COMPASS AND RULER

ON THE

BLACKBOARD.

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BY

*CHAS. DE MEDICI,

1895.

SECTION A.

Part I.

DEFINITIONS.

MATERIAL POINTS.

1. Pointed ends of anything, as, for example, a sharpened pencil point, or the points of compass-dividers, are material points.

All these things appear under the microscope as rounded surfaces. But in fact there a millions of molecules there

PUNCTS.

2. A dent made by a material point in any material substance is called a punct. Hence, puncts are given *sites* of located material points.

This dent is a hollow in a surface. Calling it a site is of doubtful propriety

DOTS.

. . .

3. Dots are symbolic marks for puncts, and represent located points.

DISTANCE.

A·   ·B

4. The intervening space between two located points marked by dots is called a distance. Hence the distance: *A B*.

LINES.

A _____ B

5. Lines are straight marks which represent distances. The line *A B* marks the distance between two given points represented by the dots *A* and *B*. Every true line marks the *shortest* distance between two points.

In Euclid lines are not necessarily straight Nor are straight lines necessarily limited

SECTION A.

MATHEMATICAL POINTS.

6. Mathematical points may be grouped into three kinds: extreme points, points of unity and intersecting points.

Not a very useful division, the two latter kinds differing little.

EXTREME POINTS.

7. The location where a given distance ends is called an extreme point. Extreme points are represented by extremities of the line which marks the distance.

This definition does not agree with the division just made

A————————B   EXHIBIT I.

A and *B* are the lineal extremities of the line *A B*, which two extremities mark the two extreme points of a given distance represented by the line *A B*.

POINTS OF UNITY.

8. When two or more extreme points represented by a number of lineal extremities blend into one common point, the common point is called a *point of unity*.
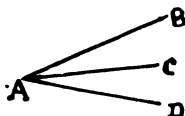
EXHIBIT 2.

A B, *A C*, *A D*, are three given lines. *A* marks the point of unity.

INTERSECTING POINTS.

9. Intersecting Points are produced when two lines cross each other.
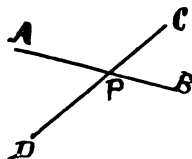
EXHIBIT 3.

The two lines *A B* and *C D* cross each other at the intersecting point *P*.

UNIFORM CURVES.

Arcs of circles (handwritten annotation)

10. Uniform Curves are described by moving one extremity of a given line while the other extremity of the same line occupies a fixed location.
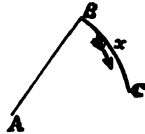
EXHIBIT 4.

A B is the given line. *A* represents the fixed location of the stationary extremity; *B* represents the moving extremity; *B x C* represents the curved course described by the motion of the extremity *B*. Curves described in this manner are uniform curves.

THE ENDLESS UNIFORM CURVE.

11. The endless uniform curve is a curve described from and to the self same point. *Circle* (handwritten annotation)
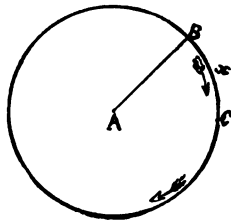
EXHIBIT 5.

It is produced by continuing the curve *B x C*, as described in Exhibit 4, until the starting point *B* is reached. This uniform endless curve marks the *longest* distance from and to the self same point, contrary to the straight line which marks the *shortest* distance between two given points. When drawn on a plane surface, the endless uniform curve is called a circle's circumference; but when the endless uniform curve is used to represent the greatest distance around a sphere or a globe, then it is called the periphery.

circumference is merely the latin imitation of the Greek periphery. Circum stands for περί, fero for φέρω (handwritten annotation)

SECTION A.
PRIME ELEMENTS.

12. The prime geometric elements are named:

<div align="center">dot   line   curve</div>

He omits superficial and solid elements –

By combinations of these prime elements the primary geometric forms are produced.

PRIMARY FORMS.

13. Geometric forms are represented by plane surfaces or by spaces bounded by geometric elements. *Perfectly arbitrary*

The primary forms are named: circles, segments and sectors.

THE CIRCLE.

14. The circle is a plane surface bounded by the circle's circumference which has already been described in Exhibit 5. The circle contains all the primary forms and every geometric form can be drawn in and evolved from the circle. *This is not so.*
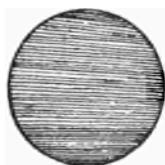
EXHIBIT 6.

THE SEGMENT.

15. The segment is formed by two elements: line and curve. Any line drawn in a circle from circumference to circumference forms a segment.

EXHIBIT 7.

The space bounded by the line *A C* and by the curve *A B C* represents a segment. When the curve of a segment is uniform, the segment is part of a circle. All segments which are parts of a circle are called *prime* segments

A useless term.

Useless term PRIME SECTORS.

16.   The prime sector is formed by two lines of equal length and one uniform curve.   The curve of the prime sector may be described with either of the two lines from a common point of unity which is located in a circle.   Lines which form parts of a sector's boundery are called legs of the sector.

EXHIBIT 8.

The lines $A\ B$ and $A\ C$ together with the curve $C\ x\ B$ form the prime sector $A\ B\ x\ C\ A$.

FIRST AXIOM.

17.   *All prime segments and sectors are contained in and are parts of a circle.*
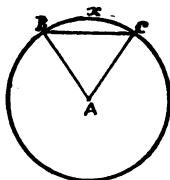
EXHIBIT 9.

For it is self evident that the sector $A\ B\ x\ C$ is contained in a circle described by the line $A\ B$ or by the line $A\ C$, and it is also self evident that the segment $C\ x\ B\ C$ is contained in the circle as well as in the given sector which is a part of the circle.

PRINCIPAL ELEMENTS.

18.   The principal elements are named: centers, radii, diameters, arcs, chords, angles and sines.

Ridiculous

SECTION A.
DEFINITIONS.

THE CENTER.

19. In any circle, the common point of unity for all the sectors in the circle is called the circle's center. This common point of unity must necessarily be *equidistant* from every part of the circle's circumference, since it is shown that the legs of sectors describe the circumference of circles. *C* marks the center in Exhibit 10.

EXHIBIT 10.

RADIUS AND RADII.

20. Lines representing legs of sectors also mark and measure the distance from center to circumference of circles. Considered as a measure of distance between a given center and a given circumference, each line is called the radius; but when two or more are spoken of and named together, they are called radii. Exhibit 10 shows five radii, either of which is the circle's radius.

SECOND AXIOM.

21. *Every sector marks a center and a circle's radius.*

THE DIAMETER.

22. Two radii of any circle extending from the circle's center in opposite directions mark the longest distance in the circle. This, the longest line in the circle, is called the circle's diameter. Hence, every diameter equals two radii as *C A* and *C B*.

ANGLES.

23. Difference in direction of radii *within* a semicircle, or, the divergence of any two given radii in a semicircle, form what is

called an angle. Exhibit 10 shows the angles $A\ C\ D$, $A\ C\ E$,
$D\ C\ E$ and $F\ C\ E$.

ARCS.

.24.   Any part of a given circumference marked off (or measured)
by a given angle is called an arc. Hence it follows, that every given
arc measures an angle and every given angle measures an arc.
Exhibit 10 and 11 show the two arcs $B\ x\ D$ and $B\ y\ F$ measured by
the angles $D\ C\ B$ and $B\ C\ F$.

CHORDS.

25.   Any line which spans a given arc is called a chord.
Exhibit 10 and 11 show the chord $E\ F$ which spans the arc $F\ B\ E$.
Now, since every arc and its chord form a segment, and as every
segment and two radii form a sector, as shown by the figure
$C\ E\ B\ F\ C$, it follows, that chords divide sectors into two parts:
one, the segment $E\ F\ B\ E$, the other, the angular figure $C\ E\ F\ C$
which is called the *angle-plane*. A line which divides the angle-
plane into equal parts is called the plane's altitude, and a line which
divides a segment into two equal parts is called the altitude of the
segment.

What is the of this word?

SINES.

26.   The altitude of an angle-plane and the altitude of a
segment contained in a given sector, produce in conjunction with the
chord which divides the sector, certain lines called *sines*.
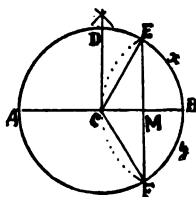

EXHIBIT 11.

That is: $E\ C\ F\ B\ E$ is the given sector; $E\ F$ is the chord which
divides the sector's angle-plane from the segment; $M\ B$ is the
altitude of the sector's segment, and $M\ C$ is the altitude of the

sector's angle-plane.   *M C*, *M B*, *M E* and *M F* represent what are
called sines.

THIRD AXIOM.

27.   *All the principal elements are contained in the circle.*

SUB-ELEMENTS.

28.   The sub-elements are evolved from the given principal
elements and are named: right angles, obtuse angles, acute angles,
perpendiculars, parallels, right sines, versed sines, co-sines, tangents
and secants.

THE RIGHT ANGLE.

29.   The right angle is obtained by dividing a semicircle into
two equal parts, as shown in Exhibit 12, by the line *C D*, which
divides the semicircle into the two equal parts : *D C A D* and *D C
B E D*. Hence, the arc-measure for a right angle is one fourth of
the circle's circumference.
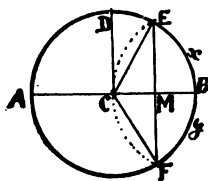
EXHIBIT 12

OBTUSE AND ACUTE ANGLES.

30.   All arcs *greater* than one fourth of a circle's circumference
measure *obtuse* angles, and all arcs *less* than one fourth of a circle's
circumference measure *acute* angles.   Hence, the angle *E C B* is
acute, and the angle *E C A* is obtuse.

PERPENDICULARS.

31.   Any two lines which form a right angle are said to be
perpendicular to each other.   Hence, the radius *C D* is perpendicular
to the diameter *A B*, and *C D* is perpendicular to *C A*.

PARALLELS.

32. When two lines are perpendicular to a given third line, the two lines are said to be parallel. *Very bad definition. A line running east and west and a line running north and south may both be perpendicular to a vertical line. Are they parallel?*
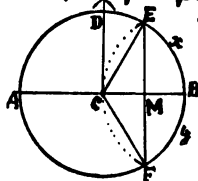
EXHIBIT 13.

The diameter *A B* is the given third line, which, with the other two lines (*C D* and *M E*) form right angles. Hence, *C D* and *M E* are parallel, because, they are both perpendicular to the common base *A B*.

THE RIGHT SINE.

33. The right sine is a given line which is perpendicular to one radius and parallel to another radius in the same circle. The line *E M* represents a right sine, because, it is perpendicular to the radius *C B* and it is parallel to the radius *C D*. *All lines in the plane of the circle satisfy this definition*

VERSED SINES AND CO-SINES.

34. "Versed sine" is the name for that distance of the radius which intervenes between the right sine and the circumference, as *M B*. (Exhibit 13.)

"Co-sine" is the name for the remaining distance of the radius intervening between the right sine and the circle's center, as *M C*. (Exhibit 13.)

TANGENTS AND SECANTS.

35. When a sine or a chord of one circle touches the circumference of another circle, such sine or chord is called a tangent. *Of all attempts to define a tangent this combines more faults than any I have ever seen.*

Thus, the chord $M B$ of the greater circle over $A B$·is tangent to the minor circle over $O D$. (Exhibit 14).
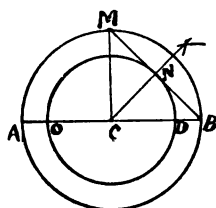
EXHIBIT 14.

When a line is drawn from the center of a minor circle and extended beyond the circumference, until it forms a point of unity with a tangent, such line is called a secant. Hence the secant $C M$.

FOURTH AXIOM.

36. *All geometric elements are contained in the circle.*

GEOMETRIC CONSTRUCTION.

37. Primary construction is effected by the use of ruler and compasses, within the compass of a circle.

38. Geometric construction is proven true when a common rule is given and applied to circles of different dimensions, and similar results are obtained.

39. The simplest process in geometric construction is called *bisecting*, which means: to divide anything given into two equal parts.

CONSTRUCTION OF THE RIGHT ANGLE
by the bi-secting process.

EXHIBIT 15.

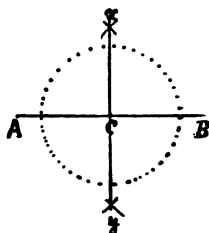**40.** *A B* is a given line. With compasses find two intersecting points as *x* and *y* which shall be equidistant from the two extreme points *A* and *B*. Draw with the ruler the bisecting line *x y* which divides the given line *A B* into two equal parts as *C B* and *C A*. This process, constructively demonstrates, that *x y* is perpendicular to *A B*. For, when from the center *C*, with any radius less than one half the line *A B*, a circle is described, it is found, that the two lines *A B* and *x y* divide the circle into four equal parts. Hence, each of the four angles formed by the process is measured by one fourth of a given circumference, which is the measure for a right angle. (See § 29.)

HOW TO FIND THE CENTER OF ANY CIRCLE.
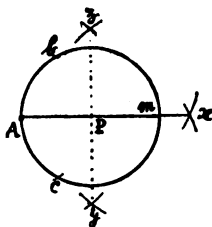
EXHIBIT 16.

41. When the circumference of a circle is given, mark a point anywhere in the circumference as at the point *A*. With compasses from *A* as a center, mark two equidistant points in the circumference as the points *b* and *c*. With compasses from *b* and *c* as centers, mark an intersecting point as *x*, equidistant from *b* and *c*; then draw a line from *x* to *A* which bisects the given circumference at *m*. Again, with compasses from *A* and *m* as centers, mark the equidistant points *z* and *y* and draw the line *z y* which bisects the diameter *A m* at *P*. Hence, *P* is the center sought.

SECTION A.

THE THREE POINT PROBLEM.

42. Any three points not in the same line are contained in a periphery, which periphery can be found by compasses and ruler.

This is not the three point problem.
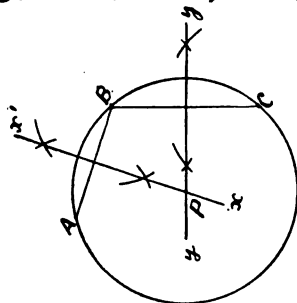

EXHIBIT 17.

A, B, and C, are three given points. Draw two lines which connect the given points, as the lines $A\,B$ and $B\,C$. Bisect these connecting lines, shown by $y\,y'$ and by $x\,x'$, and extend the bisecting lines until they intersect each other as shown by the intersecting point P. Now, from P as a center, describe a circle with a radius equal to the distance between either of the given points A, B, or C and the point P, then it is found, that a circle so described contains in its circumference the three given points.

FIFTH AXIOM.

43. *All geometric points are contained in the circle.*

SCHOLIUM.

(Explanatory Remarks.)

44. If all points not in the same line are contained in a circle's circumference, and if all points in one and the same line are contained in the *longest* line which necessarily must be a diameter of the *greatest* circle, it is self evident, that all geometric points are contained in the greatest circle. For, let the longest line be what it will, that line is still the generating factor of a circle's circumference greater in extent than itself and all-compassing as regards geometric forms and mathematical points.
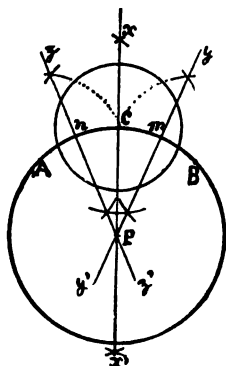
HOW TO FIND THE CIRCLE OF WHICH AN ARC IS GIVEN.

EXHIBIT 18.

45. *A C B* is a given arc. Bisect the arc *a c b* by the perpendicular line *x x'* which bisects at *c*. Describe a circle from *c* as a center over the given arc. Bisect the two arcs *b m c* and *c n a* by the lines *y y'* and *z z'* which produce the intersecting point *p*. The point *p* is the center sought and *p m* and *p n* are radii in the circle of which *A C B* is a given arc.

HOW TO CONSTRUCT PARALLEL LINES WITH COMPASSES.
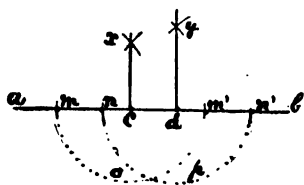
a useless problem

EXHIBIT 19.

46. On the given base line *a b* two points are given: *c* and *d*. It is required to construct two lines at the points *c* and *d* parallel to each other and perpendicular to the common base line *a b*. From *c* as a center with any radius, describe a semicircle as *m o m'* and construct the perpendicular *c x* according to rules given in § 30, and § 31. From *d* as a center with any radius, describe a semi-circle as *n p n'*. Proceed as before, and construct the second perpendicular *d y*. Then the two lines *c x* and *d y* are proven parallel, since both are perpendicular to the same base line. (See § 32.)

HOW TO PROVE SLANTING LINES PARALLEL OR NOT PARALLEL
WHEN NO BASE LINE IS GIVEN.
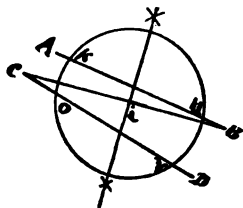
Very stupid
Any angle
EXHIBIT 20. *does as well.*

47. If two lines as A B and C D are given without a base line and it is required to prove that these two lines are or are not parallel, draw a line between the two extremities B and C and bisect that line at the point i. From i as a center, with a radius less than one half the line B C, describe a circle, then if the arcs o k and u v are equal, the lines a b and c d are parallel, but when the arcs o k and u v are unequal, as in the Exhibit 20, the lines A B and C D are not parallel.

EVOLUTION OF SECTORS AND SINES BY THE USE OF COMPASSES.
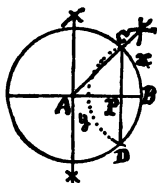
EXHIBIT 21.

THE OCTANT.

CONSTRUCTION.

48. Describe a circle. Quarter the circle. Bisect the quadrant. Then the octant A B x C A is given. With compasses from the point B as a center, and with the octant chord B C for radius, describe the curve C y D. Draw the quadrant-chord C D which intersects the radius A B at P, then three sines are given as a result of the operations: the right sine C P, the versed sine P B, the co-sine P A.

THE SEXTANT.

49. The sextant is a sector equal to one sixth of a circle.

MANNER OF CONSTRUCTION.

EXHIBIT 22.

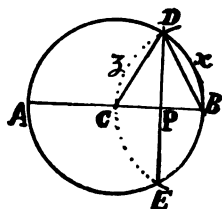Describe a circle. Draw the diameter and lay off with radius, from extreme of diameter, a chord equal to the radius. Then the sextant arc $B x D$ and the sextant chord $B D$ are given. Draw the radius $C D$, then the sextant $C B x D C$ is given. With $B D$, as a radius, from B as a center, continue the curve $D z C$ to E, and draw the tridrant-chord $D E$ which intersects the radius $C B$ at P. By these operations, three sines are given: $D P$, the right sine of a sextant, $P B$, the versed sine of a sextant, and $P C$, the co-sine of a sextant.

THE TRIDRANT. *What a word!*

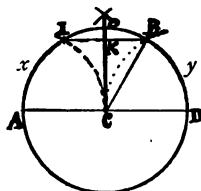**50.** The tridrant is a sector equal to one third of a circle.

EXHIBIT 23.

CONSTRUCTION.

Draw a circle and mark off with the radius a sextant arc as $D y B$. Draw the radius $C B$, then a tridrant is given and described by the figure $A C B x A$. Now, if the tridrant-arc $B x A$ is bisected at I and a sextant-chord is drawn as $B I$, and when the sextant arc

$B\ O\ I$ is bisected at O, and the radius $O\ C$ is drawn perpendicular to $A\ D$, then three sines are given: the right sine $B\ K$, the versed sine $O\ K$ and the co-sine $K\ C$, all of which are contained in the sector $B\ C\ O\ B$ which represents one twelfth of a circle. The tridrant-chord of the sector $A\ C\ B\ I\ A$ is represented by drawing a line from A to B.

SCHOLIUM.

51. Construction of quadrants and semicircles produces no sines, but the quadrant represents the sum of all sines, and the semi-circle represents the sum of all angles and sectors. The greatest sector is the semicircle less the least sector and the least sector is the difference of the greatest sector and the semicircle, which the following illustration shows.
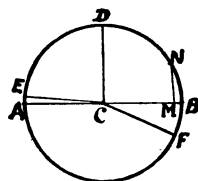

EXHIBIT 24.

If $A\ C\ E$ represents the least acute angle, then $E\ C\ B$ represents the greatest obtuse angle. And it follows, since the greatest and the least of all angles are contained in the semicircle, no arc greater than the semicircumference is required for the measurement of any and all angle-planes of sectors. Therefore, it is not proper to employ the term sector to parts of a circle greater than the semicircle. Thus, "section" $A\ C\ F\ B\ D\ E\ A$ is the better term in cases where a certain portion of a given circle is greater than the semicircle as shown in the Exhibit.

If the versed sine $M\ B$ represents the least sine, then the co-sine $M\ C$ represents the greatest sine. Now, as the radius of the circle is shown to be the sum of the least and the greatest sines, it follows, that the quadrant of any circle contains all the sines which can occur in geometric construction.
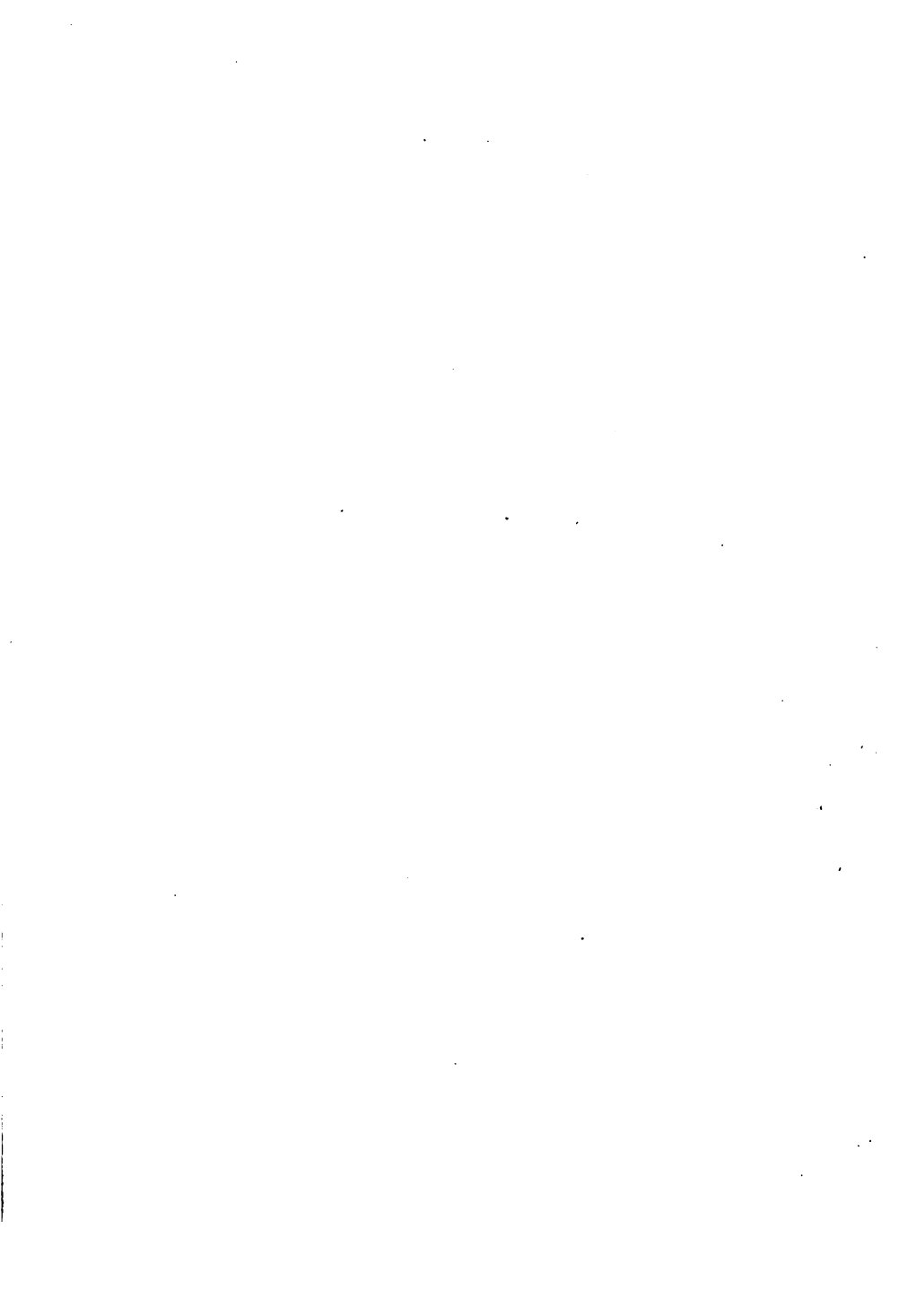
THE GEOMETRIC DIAL.

EXHIBIT 25.

52. The geometric Dial represents the summary of the 27 elements which comprise the alphabet of mathematics.

SUMMARY OF PART I, SECTION A.

THREE GEOMETRIC POINTS:
Extreme Points—Points of Unity—Intersecting Points.
THREE PRIME ELEMENTS:
Dot—Line—Curve.
THREE PRIMARY FORMS:
Circle—Sector—Segment.
EIGHT PRINCIPAL ELEMENTS:
Periphery (Circumference) — Center — Radius — Diameter.
Angle—Arc—Chord—Sines.
TEN SUB-ELEMENTS:
Right Angle—Obtuse Angle—Acute Angle—Right Sine.
Versed Sine — Co-Sine — Perpendiculars — Parallels —
Tangents—Secants.

FIVE PRIMARY SECTORS:
Semicircle—Tridrant—Quadrant—Sextant—Octant.

FIVE BASIC AXIOMS.

THE THREE POINT PROBLEM

THE GEOMETRIC DIAL.

SUPPLEMENT.

TUTORS' SCHOLIUM.

(SECTION A, PART I.)

Great Responsibility rests with the tutor who imposes on himself the task of teaching Geometry as it should be taught. The new methods of teaching elementary geometry as presented in this book are designed to popularize the science. One object aimed at is to make the study interesting to the pupil by proper object-teaching which addresses the understanding and elicits inquiry, thereby to avoid taxing the memory with information not understood. The tutor should not trust to definitions expressed by language alone. Every verbal definition should be exemplified by some tangible object, some self executed demonstration or some operative process. From first to last, such objects, demonstrations and processes are formulated in this book in a concatenated series of orderly evolved issues, which, step by step introduce and explain the necessary technical terms, without. wasting time in memorizing anything until it is required for use.

A special new feature of this Geometry is, that all geometric construction is effected *within* a circle, which keeps constantly the fact clear in the mind of the pupil, that every part of any geometric form whatever is a geometric element in some way related to the circle.